ON ASSESSMENT OF BERTALANFFY GROWTH EQUATION PARAMETERS AND INSTANTANEOUS NATURAL MORTALITY RATE OF SOUTH GEORGIA MACKEREL ICEFISH

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Abstract

An analysis and summary of extensive published and unpublished data on size-weight and size-age composition of South Georgia mackerel icefish (*Champsocephalus gunnari*) for 1965 to 1987 is presented. The results enable the calculation of reliable weight-length equations for males and females by season and the verification of parameters for the Bertalanffy growth equation. Mean value of natural mortality rate was estimated at 0.55.

Résumé

Une analyse et un résumé des données exhaustives, publiées ou non, sur la composition par poids-taille et âge-taille du poisson des glaces (*Champsocephalus gunnari*) de Géorgie du Sud sont présentés pour 1965 à 1987. Les résultats permettent le calcul d'équations poids-longueur fiables, pour les mâles et les femelles, par saison, et la vérification des paramètres de l'équation de croissance de Bertalanffy. La valeur moyenne du taux de mortalité naturelle a été estimée à 0,55.

Резюме

Представлен анализ и сводка обширных опубликованных и неопубликованных данных за 1965-1987 гг. по размерно-весовому и размерно-возрастному составу ледяной рыбы (*Champsocephalus gunnari*) в районе Южной Георгии. Результаты позволяют вывести точные уравнения взаимосвязи вес-длина для самцов и самок по сезонам, а также дают возможность выверить параметры уравнения Берталанффи. Вычисленное среднее значение коэффициента естественной смертности равняется 0,55.

Resumen

Se presenta un análisis y resumen de un gran número de datos publicados e inéditos sobre la composición de talla-peso y talla-edad del draco rayado (*Champsocephalus gunnari*) de Georgia del Sur para 1965 hasta 1987. Los resultados permiten el cálculo de razones fidedignas de peso-talla para machos y hembras por temporadas y la verificación de parámetros para la ecuación de crecimiento de Bertalanffy. El valor medio del índice de mortalidad natural se estimó en 0.55.

1. INTRODUCTION

Several papers have been published on the estimation of parameters of the Bertalanffy growth equation and instantaneous natural mortality rate for mackerel icefish (*Champsocephalus gunnari*) (Kochkin, 1985; Olsen, 1955; Kock, 1981; Linkowsky and Rembiszewsky, 1978 as cited from Kock, 1981).

Although the concept of change of natural mortality by age is generally accepted now, the problem of producing scientifically reliable assessments of pertinent coefficients has not been solved. This problem hampers the practical application of variable natural mortality rates in stock and catch assessments.

In this context, an attempt was made to determine the natural mortality rate M as a mean value, not subject to changes with age, so that this value could be used in future stock assessments.

2. MATERIALS AND METHODS

In order to calculate the parameters of weight-length relationships the 1970 to 1987 data on length and weight of *C. gunnari* were used. The results of approximately 50 000 biological analyses were considered.

The following equations were tested in order to reliably describe growth of *C. gunnari.*

$$W_t = W_\infty \{1 - e^{-K(t-t_o)}\}$$
(Bertalanffy equation)

$$Wt = W_\infty e^{-\frac{c}{a} e^{-at}}$$
(Gompertz curve)

$$W_t = \frac{W_\infty}{1 + æ^{-Kt}}$$
(logistic curve)

The method of non-linear parameter estimation (Bard, 1979) was used to define non-linear dependence parameters. The error of the observed data approximation was taken to be a criterion for selection of one or another equation and was expressed as mean-square error:

$$S^2 = \min \Sigma \frac{W_t - W_t'}{n - 1}$$

where W_t = observed value of fish weight at age t,
W_t' = weight value calculated from the equation.

Averaged data based on age determinations of 3 500 fish were used in the calculations. The age-groups which had less than three specimens in each group (primarily older age-groups) were rejected.

The methods of Baranov, Rikhter-Efanov, Beverton and Holt and Alverson-Carney were used to determine natural mortality rates, **M** (Babayan, Bulgakov et al., 1984).

The value of **M** was estimated from the age data averaged for 1965 to 1969 (unexploited population). The percentage of mature specimens by age was derived using data from biological analyses made from January to March in 1984 to 1987.

3. RESULTS AND DISCUSSION

Parameters of the weight-length equation were estimated for each season (quarter) by sex and for males and females combined. As is evident from Table 1, certain differences exist between parameters for the females and males, namely, the females have a somewhat larger weight. The change of the parameter **b** in the weight-length equation by season is related to changes in the fish condition, amount of food in the stomach and gonad maturity state. The largest values of the parameter **b** correspond to pre-spawning, spawning and pre-wintering periods. As the **b** value considerably differ from 3.0 (3.204-3.572), the use of equations with the assumed isometric dependence should be avoided.

Our estimates are similar to those of Kock (1981) ($W=0.0017\ L^{3.3627}$), but differ from estimations of Linkowsky and Rembiszewski (1978) ($W=0.0515\ L^{2.554}$), which can be evidently attributed to a small set of data used in the latter estimations (n=75).

As mentioned above, several curves were used to estimate the growth equation parameters. As is evident from Table 2, the results gained from the use of the logistic curve and Gompertz curve appeared to be unacceptable since values of **k** and W_∞ are obviously inflated. The method of Hohendorf for the linear growth yielded good results, however, W_∞ from this method also appeard to be much larger than that observed (W_{max}=2 750). Mean long-term data on the length and weight by age were the initial data used in the calculation (Table 3).

In terms of the least approximation error, the best results were obtained for the Bertalanffy growth equation using the method of non-linear estimation with some samples rejected (data for the older age-group were rejected because of their paucity and, hence, unreliability).

The analysis of data presented in Table 4 shows that there is a close agreement between authors of all growth parameters except those estimated by Olsen. The growth coefficient **k** defined by other authors ranges from 0.1202 to 0.1478. The value of **k** calculated by Kochkin fits the same range, and the values estimated by Kock (1981) approach the upper range limit.

The parameter **k** shows at what rate the fish achieves its maximum length. As L_∞ calculated by Kock (1981) is somewhat lower than our value, his value for **k** appears to be higher. Considering that the maximum observed length of the fish in the study area is 69 cm (i.e. very close to our estimation of L_∞=68.8) k=0.1340 is used for further calculations.

No reliable rates of instantaneous mortality have been found as yet. Kock (1981) reported two quite different values: 0.61 estimated by Rikhter-Efanov method and 0.22 estimated by Pauly method. As previously mentioned, the averaged data for 1965 to 1969 were used by the authors for estimating **M**. These data are presented below.

Age	1	2	3	4	5	6	7	8
n	20	258	509	272	227	119	49	15
lnn	2.99	5.55	6.23	5.60	5.42	4.78	3.89	2.70
$lnn_{i+1} - lnn_i$	2.56	0.68	-0.62	-0.19	-0.64	-0.89	-1.19	-

For calculation according to the Baranov method, age-groups 3 to 7 were used:

$$M = \frac{0.62 + 0.19 + 0.64 + 0.89}{4} = 0.58$$

The resultant mean square error of this value will be 0.31.

Two values of M based on Beverton and Holt method are as follows:

$$M_1 = \frac{1}{\bar{t} - t'} = \frac{1}{4.67 - 3.0} = 0.60$$

where $\bar{t}$ = mean age of year classes fully represented in a sample
t' = age, beginning from which year classes begin to be fully represented in a sample,

$$M_2 = \frac{L_\infty - \bar{T}\,K}{(\bar{T} - l')} = \frac{(68.90-35.31) \cdot 0.134}{35.31 - 26.55} = 0.51$$

where $\bar{T}$ = mean length of fish fully represented in a sample,
l' = length of fish, beginning from which year classes are fully represented in a sample,
L_∞ = maximum length.

The value of **M** estimated by Rikhter-Efanov method is as follows:

$$M = \frac{1.521}{t_m 0.72} - 0.155 = \frac{1.521}{3.0^{0.72}} - 0.155 = 0.54$$

where t_m = age at massive maturation (as shown by our studies, 90% of the fish attain sexual maturity at the age of 3).

The value of **M** estimated by the Alverson-Carney method was 0.32.

$$M = \frac{3K}{e^{T \cdot K} - 1} = \frac{3 \cdot 0.134}{e^{6 \cdot 0.134} - 1} = 0.32$$

where T = time of biomass culmination estimated by the authors at 6 years.

The variety of **M** values calculated by these different methods is presented in the table below.

Name of Method	Value of M
Rikhter-Efanov (1976)	0.54
Baranov	0.58
Beverton and Holt, by age	0.60
Beverton and Holt, by length	0.51
Alverson-Carney	0.32

Most of the resultant values of **M** are close to and have $\bar{M}$=0.55. One exception is M=0.60 calculated by Beverton and Holt method.

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Table 1: Weight-length ratio parameters for *C. gunnari* by sex and season ($W=aL^b$).

Quarter of the year	Combined Data a	b	Males a	b	Females a	b
1	0.0001935	3.372	0.003446	3.204	0.002221	3.338
2	0.001510	3.424	0.001026	3.528	0.000998	3.552
3	0.000947	3.548	0.001004	3.530	0.000908	3.574
4	0.002212	3.299	0.04558	3.299	0.002184	3.302

Table 2: Parameters of different growth equations for *C. gunnari*: linear and weight growth.

Growth Equation (Method of Estimation)	Parameters and their mean square errors (in brackets) for Linear Growth Equation			Parameters and theirmean square eErrors (in brackets) for Weight Growth Equation		
	$L\infty$ (cm)	t_o	K	$W\infty$ (g)	t_o	K
Bertalanffy equation (Hohendorf method)	71.20	0.1548	0.1354	5287.5	0.8656	0.0793
Logistic curve (Nonlinear regression analysis)				3509.9 (945.2)	0.2349 (0.050)	28.0539 (7.936)
Gompertz curve (Nonlinear regression analysis)				5107.1 (228.3)	0.1028 (0.036)	0.4709 (0.209)
Bertalanffy equation (Nonlinear regression analysis)	68.8 2.91	0.28 0.19	0.1340 0.0140	3276.1 (708.6)	0.180 (0.874)	0.12 (0.028)

Table 3: Mean long-term length and weight values for *C. gunnari* by age.

Age	1	2	3	4	5	6	7	8	9	12	15
Weight (g)	2.7	24.8	77.6	163.1	227.8	416.4	572.4	739.7	912.9	1427.2	1881.6
Length (cm)	10.8	18.1	24.5	30.0	34.9	39.1	42.8	46.1	49.0	55.5	49.9

Table 4: Parameters of linear growth for *C. gunnari* calculated by different authors.

Parameters	Olsen, 1955 Ford-Walford Method	Kock, 1981 Ford-Walford Method	Kock, 1981 Nonlinear Regression Method	Kochkin, 1985 Hohendorf Method	Our Estimates Nonlinear Regression Method
K	0.3978	0.1570	0.1528	0.1296	0.1340±0.0138
L_∞	42.10	65.10	64.30	71.94	68.9±2.9
t_o	- 1.3557	0.3849	0.2673	- 0.6722	-0.2798±0.19

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